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MATHEMATICAL MODEL FOR FINDING THE GALLBLADDER WALL THICKNESS

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ABSTRACT

Gallbladder wall thickening and impaired contractility are currently reported in cirrhotic patients and often related to portal hypertension and hepatic failure. The purpose of this work was to evaluate, by using standard Brownian motion process with random walk method, gallbladder wall thickness and gallbladder emptying after a standard meal in normal subjects and in patients with compensated liver cirrhosis without gallstones.

INTRODUCTION

Gallbladder wall thickening and impaired contractility are currently reported in cirrhotic patients, but most published studies were carried out in patients with portal hypertension and hepatic failure [2] & [6]. Diffuse gallbladder wall thickening is a nonspecific alteration caused by several disorders, including both intrinsic diseases and extra cholecystic diseases, such as acute hepatitis, liver cirrhosis, hypoalbuminemia, congestive heart failure, acquired immunodeficiency syndrome, pancreatitis, myeloma, and acute pyelonephritis. Gallbladder emptying in response to a meal is a physiological phenomenon, mainly coordinated by the rate of gastric emptying of foods in duodenum and by the subsequent release of cholecystokinin (CCK), which triggers, gallbladder contraction. In normal subjects gallbladder emptying is affected by several factors: age, body surface area, wall thickness, fasting volume, hormonal factors, and meal composition. Impaired gallbladder contractility has been suggested to increase the incidence of gallstones in cirrhotic patients, although incongruous results are reported [6] & [7].

Real time ultrasound (US) is the method used for direct gallbladder visualization under physiological and pathological conditions, since it allows repeated measurements at short intervals and provides information for the study of gallbladder wall thickness, content, and contraction. Ultrasound assessment of spleen dimensions and portal vein diameter is a useful noninvasive method able to lead to the diagnosis of portal hypertension. Moreover, echodoppler flowmetry is able to quantitatively assess portal flow and mean blood velocity. A significant correlation between the reduction in portal flow velocity and the severity of the disease, evaluated by the Child Pugh score is reported in literature [7].

Let $\{A(t), t \geq 0\}$ be a standard Brownian motion process. We have $P\{A(t) \leq x\} = \Psi\left(x/\sqrt{t}\right)$ for $t > 0$ where $\Psi(x)$ is the normal distribution function from the random walk $\{A_a, a \geq 0\}$ of Brownian motion process. If $b \geq 1$ and $c \geq 1$, let us define ® as the smallest $n = 0,1,2,\ldots\ldots\ldots\ldots\ldots$ for which either $A_a = b$ or $A_a = -c$. We can interpret ® as the duration of games in the classical ruin problem. We have from the random walk of standard Brownian motion,

$$G\{\omega^{®}\} = \frac{[B]^b + [B]^c}{1 + [B]^{b+c}}$$

If $|\omega| \leq 1$ where B is defined by $B = \varphi(\omega) = \frac{(1-\sqrt{1-\omega^2})}{\omega}$ and it is fitted with uniform distribution.

MATHEMATICAL MODEL

Let $\{A(t), t \geq 0\}$ be a standard Brownian motion process. We have $P\{A(t) \leq x\} = \Psi\left(x/\sqrt{t}\right)$ for $t > 0$ where

$$\Psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2}\, du \qquad (1)$$

is the normal distribution function.

Let us recall some results for $\{A_a, a \geq 0\}$ which we need in this paper. See [4], We have

$$P\{A_n = 2j - n\} = \binom{n}{j} \frac{1}{2^n}$$

for $j = 0,1,2,\ldots\ldots\ldots, n$ and by the central limit theorem

$$\lim_{n \to \infty} P\left\{\frac{A_n}{\sqrt{n}} \leq x\right\} = \Psi(x)$$

where $\Psi(x)$ is defined by (1).

Let us define $\beta(b)$ as the first passage time through b $(b = 0, \pm1, \pm2, \ldots\ldots\ldots$ that is,

$$\beta(b) = inf\{a : A_a = b \text{ and } a \geq 0\}$$

We have

$$P\{\beta(b) = b + 2j\} = \frac{b}{b+2j}\binom{b+2j}{j}\frac{1}{2^{b+2j}} \quad (2)$$

for $b \geq 1$ and $j \geq 0$. If $1 \leq b \leq n$, then

$$P\{\beta(b) \leq n\} = P\{A_n \geq b\} + P\{A_n > b\} \quad (3)$$

By (2),

$$\sum_{n=0}^{\infty} P\{\beta(b) = n\}\omega^n = [\varphi(\omega)]^b = B^b$$

where $B = \varphi(\omega)$, for $b \geq 1$ and $|\omega| \leq 1$ where $\varphi(0) = 0$ and

$$B = \varphi(\omega) = \frac{(1-\sqrt{1-\omega^2})}{\omega} \quad (4)$$

for $0 < |\omega| \leq 1$. The identity

$$\sum_{j=0}^{n} P\{\beta(b) = j\} P\{\beta(b) = n - j\} = P\{\beta(b + c) = n\} \quad (5)$$

is valid for any $b \geq 1, c \geq 1$ and $n \geq 1$.

We note that

$$P\{\beta(1) = 2n + 1\} = \frac{D_n}{2^{2n+1}} \quad (6)$$

for $n = 0,1,2,\ldots\ldots\ldots\ldots\ldots$ where

$$D_n = \binom{2n}{n}\frac{1}{n+1}$$

is the n^th^ Catalan number. Let us define

$$P(n, v) = \sum_{\substack{\alpha_1+\alpha_2+\cdots+\alpha_n = v \\ \alpha_1+2\alpha_2+\cdots+n\alpha_n = n}} \frac{v!}{\alpha_1!\alpha_2!\ldots\alpha_n!} D_0^{\alpha_1} D_1^{\alpha_2} \ldots\ldots\ldots\ldots D_{n-1}^{\alpha_n} \qquad (7)$$

for $1 \leq v \leq n$. To evaluate (7) let us express each Catalan number in (7) by (6). By the repeated applications of (5) we obtain that

$$P(n, v) = 2^{2n-v} P\{\beta(v) = 2n - v\} = \binom{2n-v}{n}\frac{v}{2n-v}$$

By (3) we obtain that

$$\sum_{s=j}^{r} P\{\beta(2j) = 2s\} = P\{\beta(2j) \leq 2n + 1\}$$

$$= 2P\{A_{2a+1} \geq 2j + 1\} = \sum_{s=j}^{a} \binom{2a+1}{a-s}\frac{1}{2^{2a}}$$

for $j = 0,1,2,\ldots\ldots\ldots\ldots\ldots a$.

If $b \geq 1$ and $c \geq 1$, let us define ® as the smallest $n = 0,1,2,\ldots\ldots\ldots\ldots\ldots$ for which either $A_a = b$ or $A_a = -c$. We can interpret ® as the duration of games in the classical ruin problem. See [3]. By the results of [1], we have

$$G\{\omega^{®}\} = \frac{[B]^{b}+[B]^{c}}{1+[B]^{b+c}} \quad (8)$$

If $|\omega| \leq 1$ where B is defined by (4). See also [5].

EXAMPLE

The study involved 23 consecutive patients (13 males and 10 females; mean age 54.5 years, range 42–67; mean BMI 23.8 kg/m2, range 18.2–26) affected by Child Pugh class A liver cirrhosis without gallstones. The diagnosis of liver cirrhosis was made on the basis of clinical and biochemical features and confirmed by histological examination. Abdominal ultrasound and gastrointestinal endoscopy were performed in order to exclude the presence of splenomegaly, subclinical ascites, gastroesophageal varices, and portal hypertensive gastropathy.

A single operator performed all the ultrasound examinations both in cirrhotic patients and in controls. Real time bidimensional and doppler ultrasound examinations were performed using a 3.5MHz transducer. Gallbladder was examined by means of the images obtained in both supine and left lateral decubitus in order to evaluate wall thickness, longest axis, width, and depth; portal vein was studied in supine decubitus and suspended respiration. We measured on the morning of the test, after an overnight fast, basal measurements of gallbladder wall thickness (GWT), portal vein diameter (PD), portal cross sectional area (CSA), mean portal velocity (PV), portal vein flow (PVF), and gallbladder fasting volume (FV) were taken in all subjects.

Figure 1. Gallbladder emptying curves after the test meal in Child Pugh class A cirrhotic patients and in healthy controls

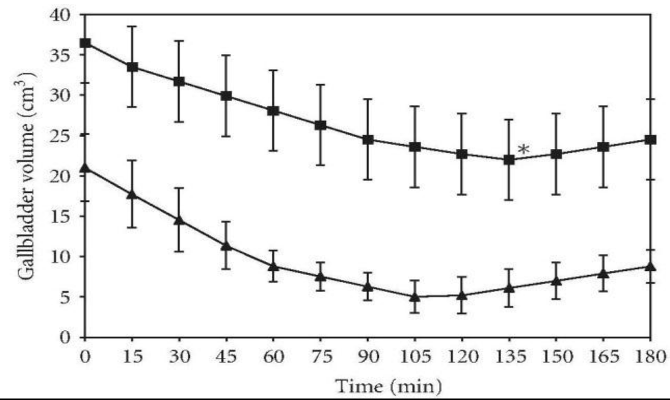
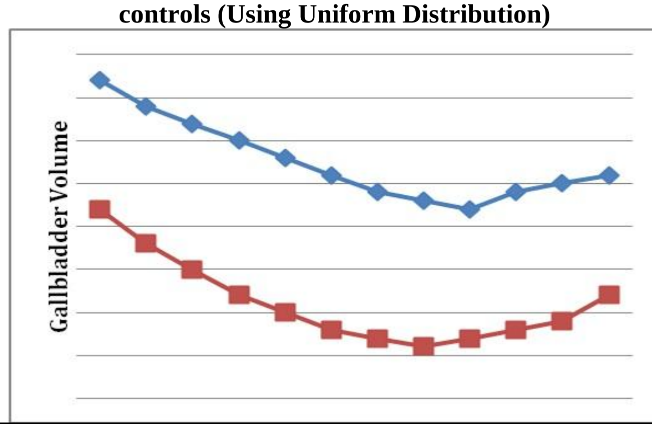


Figure 2. Gallbladder emptying curves after the test meal in Child Pugh class A cirrhotic patients and in healthy controls (Using Uniform Distribution)



CONCLUSION

The random walk of Brownian motion process gives the same as the medical report. There is no significance difference between medical and mathematical reports. The medical reports are beautifully fitted with the mathematical model. Hence the mathematical report {Figure (2)} is coinciding with the medical report {Figure (1)}.

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